


THEORY OF COMPUTATION AND COMPILERS

Unit - II

CONTEXT FREE GRAMMARS AND PARSING

- Introduction
- Context-Free Grammars - Derivation, Parse trees, Ambiguity
- Types of Parsers
- LL(K) grammars and LL(1) parsing
- Bottom-up Parsing - handle pruning
- **LR Grammar Parsing** 
- LALR parsing
- Parsing ambiguous grammars
- Error Recovery in Parsing
- YACC programming specification

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Unit-II: Syntax Analysis (or) Parser

Introduction to LR Parsing

Outline:

- Introduction
- Why LR Parsers
- Items and the LR(0) Automaton

LR Parsing

Introduction:

The **LR(k)** parser will parse the input string by applying **rightmost derivation in reverse**. The **LR(k)** parsers are **bottom-up parsers** that are capable of handling **context-free languages** very efficiently.

where

- The **L** is for **left-to-right scanning of the input**.
- The **R** for constructing a **rightmost derivation in reverse**,
- The **k** for the **number of input symbols of lookahead** that are used in making parsing decisions.

The cases **k = 0** or **k = 1** are of practical interest, and we shall only consider **LR** parsers with **k ≤ 1** here. When (**k**) is omitted, **k** is assumed to be **1**.

Why LR Parsers

- **LR** parsers are **table-driven**, much like the **nonrecursive LL** parsers.
- A **grammar** for which we can construct a **parsing table** using one of the **LR** parsing methods (**SLR**- Simple LR / **CLR**- Canonical LR / **LALR**- Look-Ahead LR) is said to be an **LR grammar**.
- For a grammar to be **LR** it is sufficient that a **left-to-right shift-reduce parser** be able to recognize **handles** of **right-sentential forms** when they appear on **top of the stack**.

Why LR Parsers

LR parsing Advantages:

LR parsing *is attractive for a variety of reasons:*

- **LR parsers** can be constructed to recognize virtually all **programming language constructs** for which **context-free grammars** can be written.
- The **LR-parsing** method is the most general **nonbacktracking shift-reduce parsing** method known, yet it can be implemented as efficiently as other, more **primitive shift-reduce** methods.

Why LR Parsers

LR parsing Advantages:

- An **LR parser** can detect a **syntactic error** as soon as it is possible to do so on a **left-to-right scan of the input**.
- **LR parsers** can describe more languages than **LL grammars**.

Why LR Parsers

LR parsing Disadvantages:

- The main **drawback** of **LR parsers** is that they consume **too much time to construct** by hand for a typical **programming language grammar**.
- But, this **disadvantage can be eliminated** using **LR parser generators** such as **YACC** (**Yet-Another Compiler-Compiler**).

Types of LR Parsers

There are **three types** of **LR parsers** are shown below:

- 1. Simple LR parsers (SLR) :** This parser is easy to implement. Hence the name **simple LR**. But, sometimes it may fail to produce the parsing table for certain types of grammars.
- 2. Canonical LR parsers (CLR) :** This parser is more powerful and works on very large class of grammars. But, it is very expensive to implement.

Types of LR Parsers

There are **three types** of **LR parsers** are shown below:

3. Look-Ahead LR parsers (LALR) : This parser is intermediate in power between **SLR** and **CLR** parsers. This parser can be implemented for most of the grammars.

Items and the LR(0) Automaton

Definition: Item or LR(0) item

An LR(0) item or item of a grammar G is a production of G with a dot(.) on the Right-Hand Side (RHS) of a production.

Example:

Consider the production $A \rightarrow BCD$. The various items that can be obtained using this production are:

$A \rightarrow .BCD$

$A \rightarrow B.CD$

$A \rightarrow BC.D$

$A \rightarrow BCD.$

Items and the LR(0) Automaton

Note:

For the production of the form $A \rightarrow \epsilon$. The item is:

$A \rightarrow \cdot$

Using these **items** called **canonical LR(0)** collection, we can construct a **DFA** (**Deterministic Finite Automata**) which is used to make **parsing decisions** such as **shifting** and **reducing**.

Items and the LR(0) Automaton

The **canonical LR(0)** collection of items for a grammar can be obtained using:

- An **augmented grammar**
- **CLOSURE** function
- **GOTO** function
- **ITEMS** function

Items and the LR(0) Automaton

Augmented grammar:

Let $G = (V, T, S, P)$ be grammar. An **augmented grammar** G' for the grammar G is defined as:

$$G' = (V', T, S', P')$$

where

$$V' = V \cup S'$$

$$P' : P \cup \{S' \rightarrow S\}$$

S' is the **start symbol** for the **augmented grammar** G' .

Items and the LR(0) Automaton

Augmented grammar:

The main purpose of the **new starting production** is to indicate to the **parser** when it should **stop parsing** and **announce acceptance of the input**. That is, **acceptance** occurs whenever the **parser** is about to **reduce** using the production $S' \rightarrow S$.

Items and the LR(0) Automaton

Augmented grammar:

Example:

Obtain the augmented grammar for the following grammar:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

Items and the LR(0) Automaton

Augmented grammar:

Example: *Solution*

By adding the production $E' \rightarrow E$ to the given grammar G , we get the augmented grammar G' as shown below:

$$E' \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid id$$

Items and the LR(0) Automaton

Closure of Item Sets:

If I is a set of *items* for a grammar G , then $CLOSURE(I)$ is constructed using the following two rules:

Rule 1: Initially, add every *item* in I to $CLOSURE(I)$.

Rule 2: If $A \rightarrow \alpha.B\beta$ is in $CLOSURE(I)$ and $B \rightarrow \gamma$ is a production and $B \rightarrow .\gamma$ is not there in $CLOSURE(I)$, then add $B \rightarrow .\gamma$ to $CLOSURE(I)$.

Rule 3: Apply **rule 2** repeatedly until no more new items are added to $CLOSURE(I)$.

Items and the LR(0) Automaton

Closure of Item Sets:

Example:

Consider the following augmented grammar:

$E' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

a) If $I = \{E' \rightarrow \cdot E\}$ then find **CLOSURE (I)**

b) If $I = \{F \rightarrow (\cdot E)\}$ then find **CLOSURE (I)**

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

The **CLOSURE(I)** can be obtained using the given productions as shown below:

- a) Since $I = \{E' \rightarrow .E\}$ add $E' \rightarrow .E$ to **CLOSURE(I)** and compute **CLOSURE(I)** as shown below:

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

CLOSURE($E' \rightarrow .E$) =

{

$E' \rightarrow .E$ placed initially. After **dot**(.), **E** is present. So, add **E-productions** beginning with **dot** as shown below:

$E \rightarrow .E + T$

$E \rightarrow .T$

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

CLOSURE($E' \rightarrow \cdot E$) =

{

$E \rightarrow \cdot T$ After **dot**(.), **T** is present. So, add **T-productions** beginning with **dot** as shown below:

$T \rightarrow \cdot T * F$

$T \rightarrow \cdot F$

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

CLOSURE($E' \rightarrow .E$) =

{

$T \rightarrow .F$ After **dot**(.), **F** is present. So, add **F-productions** beginning with **dot** as shown below:

$F \rightarrow .(E)$

$F \rightarrow .id$ After **dot**(.) no variable (or non-terminal) is present. So, stop at this point.

}

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

b) Since $I = \{F \rightarrow (.E)\}$ add $F \rightarrow (.E)$ to **CLOSURE(I)** as shown below:

CLOSURE($F \rightarrow (.E)$) =

{

$F \rightarrow (.E)$ After **dot**(.), **E** is present. So, add **E-productions** beginning with **dot** as shown below:

$E \rightarrow .E + T$

$E \rightarrow .T$

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

b) Since $I = \{F \rightarrow (.E)\}$ add $F \rightarrow (.E)$ to **CLOSURE**(I) as shown below:

CLOSURE($F \rightarrow (.E)$) =

{

$E \rightarrow .T$ After **dot**(.), **T** is present. So, add **T-productions** beginning with **dot** as shown below:

$T \rightarrow .T * F$

$T \rightarrow .F$

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

b) Since $I = \{F \rightarrow (.E)\}$ add $F \rightarrow (.E)$ to **CLOSURE**(I) as shown below:

CLOSURE($F \rightarrow (.E)$) =

{

$T \rightarrow .F$ After **dot**(.), **F** is present. So, add **F-productions** beginning with **dot** as shown below:

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Closure of Item Sets:

Example: *Solution*

- b) Since $I = \{F \rightarrow (.E)\}$ add $F \rightarrow (.E)$ to **CLOSURE(I)** as shown below:

CLOSURE($F \rightarrow (.E)$) =

{

$F \rightarrow .id$ After **dot**(.) no variable (or non-terminal) is present. So, stop at this point.

}

Items and the LR(0) Automaton

Algorithm to find CLOSURE(I) :

Function CLOSURE(I)

{

J = I

repeat

for each item $A \rightarrow \alpha \cdot B \beta$ is in J and for each production $B \rightarrow \gamma$ in G

if $B \rightarrow \cdot \gamma$ is not in J then

add $B \rightarrow \cdot \gamma$ to J

return J

}

Items and the LR(0) Automaton

GOTO function:

The **GOTO** (**I**, **X**) is a function which accepts set of items **I** as the *first argument* and **X** which is a *grammar symbol* (it can be either a **terminal** or a **non-terminal**) as the *second argument*. It is formally defined as shown below:

If **A** \rightarrow **α .X β** in **I**

GOTO (**I**, **X**) = **CLOSURE** (**A** \rightarrow **α X. β**)

Items and the LR(0) Automaton

GOTO function:

The **GOTO** function is used to define the *transition* from *one set of items* to *other set of items*. Using these transitions we can easily obtain the **parsing table** consisting of **ACTION** and **GOTO**. **I** is a *set of items* for a grammar **G**.

Items and the LR(0) Automaton

Algorithm to find GOTO (I, X) :

Function **GOTO** (I, X)

{

if (**A** \rightarrow **α .X β** is in I)

return **CLOSURE** (**A** \rightarrow **α X. β**)

}

Items and the LR(0) Automaton

GOTO function:

Example:

Consider the following augmented grammar:

$E' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

If $I = \{ [E \rightarrow T.] \}$, $\{ [E \rightarrow E.+T] \}$ then compute **GOTO(I, +)**

Items and the LR(0) Automaton

GOTO function:

Example: Solution

Given that $I = \{ [E \rightarrow T.] \}, \{ [E \rightarrow E.+T] \}$.

$E \rightarrow T.$ is already completed item because **dot** (.) is placed at the end.

In $E \rightarrow E.+T$ Immediately after **dot** (.) there is a symbol '+'. So, to compute **GOTO** (I, +):

- Let us consider the item: $E \rightarrow E.+T$
- Shift **dot** (.) to next position: $E \rightarrow E+.T$
- Now, it is necessary to compute: **CLOSURE** ($E \rightarrow E+.T$)

Items and the LR(0) Automaton

GOTO function:

Example: *Solution*

GOTO (**I**, **+**) = **CLOSURE** (**E** \rightarrow **E+.** **T**) =

{

E \rightarrow **E+.** **T** Placed initially. After **dot** (.) **T** is present. So, add **T**-productions beginning with **dot** (.) as shown below:

T \rightarrow **.** **T** * **F**

T \rightarrow **.** **F**

Items and the LR(0) Automaton

GOTO function:

Example: Solution

GOTO (**I**, **+**) = **CLOSURE** (**E** → **E+ . T**) =

{

T → **. F** After **dot** (**.**) **F** is present. So, add **F**-productions beginning with **dot** (**.**) as shown below:

F → **. (E)**

F → **. id** After **dot** (**.**) no non-terminal is present. So, stop at this point.

}

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

The canonical collection of sets of LR(0) items are computed using the following algorithm:

Function **ITEMS** (G')

{

$C = \text{CLOSURE}(S' \rightarrow .S)$

repeat

 for each set of items I in C

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

Function **ITEMS** (G')

{
 $C = \text{CLOSURE}(S' \rightarrow \cdot S)$

repeat

 for each set of items I in C

 for each grammar symbol X

 if (**GOTO** (I, X) is not empty and not in C

 add **GOTO** (I, X) to C

until no more sets of items are added to C ;

return C

}

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

Example:

Consider the following augmented grammar:

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid id$$

Obtain **LR(0)** items (or) compute **canonical collection of sets** of **LR(0)** items.

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

Example: *Solution*

Given grammar:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

Augmented grammar:

$E' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

Example: The various items obtained along with **GOTO**'s are obtained by constructing the initial item with **CLOSURE** ($E' \rightarrow .E$) as shown below:

$I_0:$

$E' \rightarrow .E$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

I_1 : GOTO (I_0 , E)

$E' \rightarrow E.$

$E \rightarrow E. + T$

I_2 : GOTO (I_0 , T)

$E \rightarrow T.$

$T \rightarrow T. * F$

I_3 : GOTO (I_0 , F)

$T \rightarrow F.$

I_4 : GOTO (I_0 , $()$)

$F \rightarrow (.E)$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

I₅: GOTO (I₀, id)

F → id.

I₆: GOTO (I₁, +)

E → E + . T

T → . T * F

T → . F

F → . (E)

F → . id

I₇: GOTO (I₂, *)

T → T * . F

F → . (E)

F → . id

I₈: GOTO (I₄, E)

F → (E .)

E → E . + T

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

$I_9: \text{GOTO}(I_4, T) = I_2$

$E \rightarrow T.$

$T \rightarrow T * F$

$I_{10}: \text{GOTO}(I_4, F) = I_3$

$T \rightarrow F.$

$I_{11}: \text{GOTO}(I_4, () = I_4$

$F \rightarrow (.E)$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

I_{12} : GOTO (I_4 , id) = I_5

$F \rightarrow id.$

I_{13} : GOTO (I_6 , T)

$E \rightarrow E + T.$

$T \rightarrow T * F$

I_{14} : GOTO (I_6 , F) = I_3

$T \rightarrow F.$

I_{15} : GOTO (I_6 , id) = I_5

$F \rightarrow id.$

I_{16} : GOTO (I_6 , $()$) = I_4

$F \rightarrow (.E)$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

I_{17} : GOTO (I_7 , F)

$T \rightarrow T * F.$

I_{18} : GOTO (I_7 , $()$) = I_4

$F \rightarrow (.E)$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

I_{19} : GOTO (I_7 , id) = I_5

$F \rightarrow id.$

I_{20} : GOTO (I_8 , $)$)

$F \rightarrow (E).$

I_{21} : GOTO (I_8 , $+$) = I_6

$E \rightarrow E +.T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

$I_{22}: \text{GOTO}(I_{13}, *) = I_7$

$T \rightarrow T * . F$

$F \rightarrow . (E)$

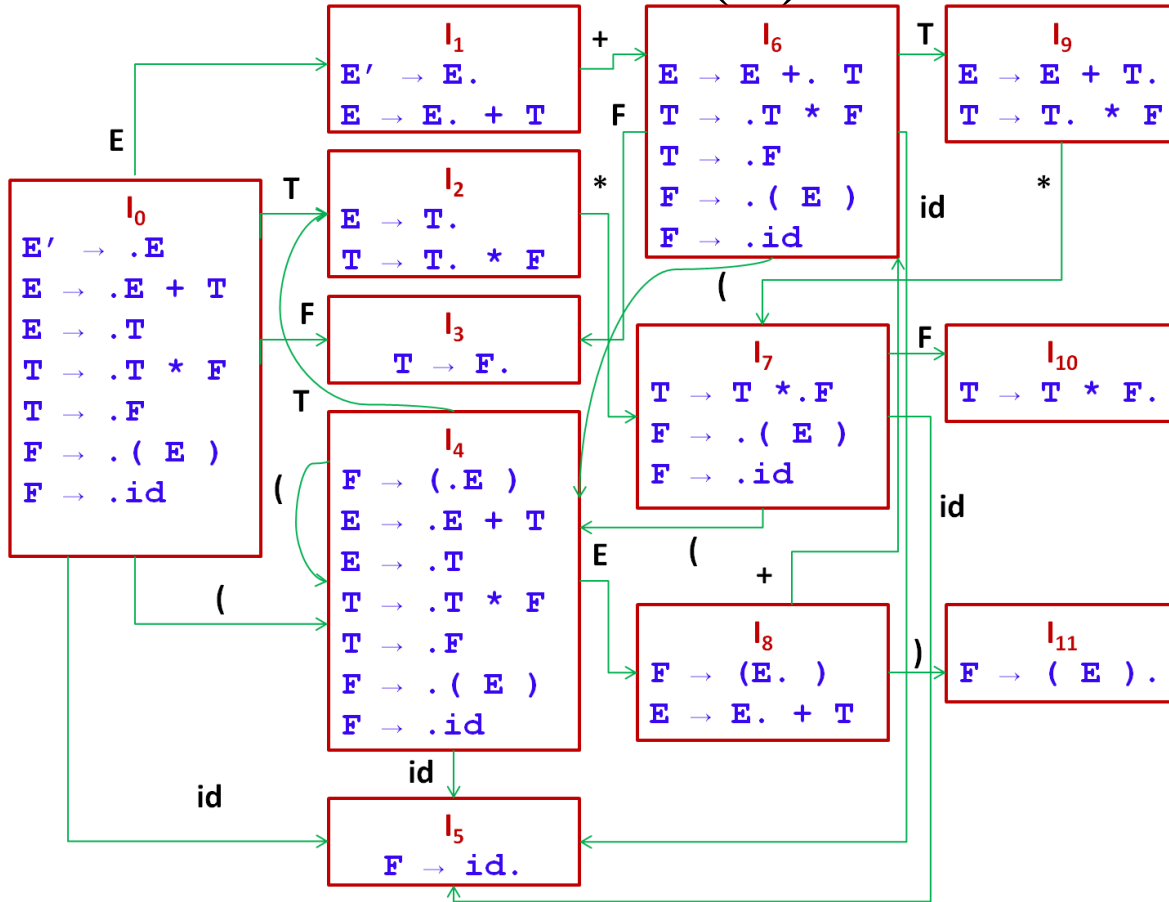
$F \rightarrow . id$

Items and the LR(0) Automaton

Construction of DFA for Canonical collection of sets of LR(0) items:

Items and the LR(0) Automaton

Fig: DFA for LR(0) items



Items and the LR(0) Automaton

Note: All the items called **LR(0)** items in the above Automaton (**DFA**) are divided into two classes:

- 1. Kernel items:** The initial item, $S' \rightarrow .S$ and all the items that are not starting with **dot** (.) are called **kernel items**.
- 2. Non-kernel items:** All the items with **dot** (.) at the beginning except the item $S' \rightarrow .S$ are called **non-kernel items**.

Items and the LR(0) Automaton

Canonical collection of sets of LR(0) items:

Practice problem:

Construct the LR(0) items for the following grammar:

$S \rightarrow L = R \mid R$

$L \rightarrow * R \mid id$

$R \rightarrow L$

Check whether the above grammar is LR(0) grammar or not

[JNTUH: March/April - 2021]

Summary...

Bottom-Up Parsing:

- Introduction
- Why LR Parsers
- Items and the LR(0) Automaton

Reading: Aho2, Section 4.6.1 & 4.6.2

Next Lecture: The LR-Parsing Algorithm