

Unit - I | Lecture- 08

Eliminating ϵ -transitions

(Equivalence of NFA with and without ϵ -moves)

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Finite Automata with Epsilon-Transitions(NFA- ϵ)

- Eliminating ϵ -Transitions

(Conversion of NFA with ϵ -transitions to NFA without ϵ -transitions)

- Example Problems

Equivalence of NFA with and without ϵ -moves:

Theorem:

For every **NFA with ϵ -moves**, there exist an equivalent **NFA without ϵ - moves**.

Equivalence of NFA with and without ϵ -moves:

Procedure:

Let $M = (Q, \Sigma, \delta, q_0, F)$ be **NFA- ϵ** and

Let $M' = (Q, \Sigma, \delta', q_0, F')$ be an equivalent **NFA without ϵ -moves**.

where

$$F' = \begin{cases} F \cup \{q_0\}, & \text{if } \epsilon\text{-closure}(q_0) \text{ contains a final state} \\ F, & \text{Otherwise} \end{cases}$$

$$\delta'(q, a) = \delta^{\wedge}(q, a) = \epsilon\text{-closure}(\delta(\delta^{\wedge}(q, \epsilon), a))$$

for every $q \in Q$ and for every $a \in \Sigma$

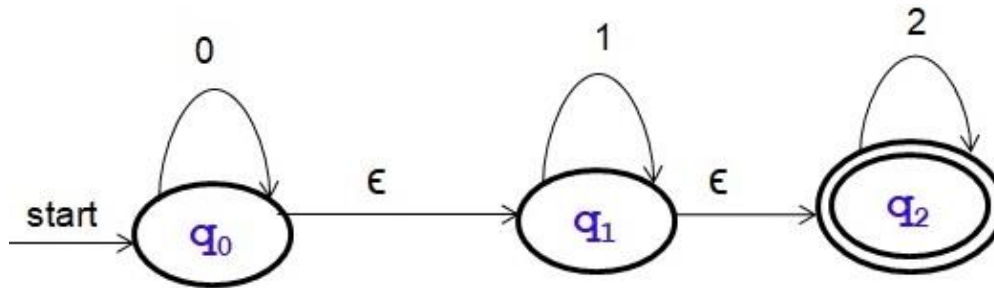
Equivalence of NFA with and without ϵ -moves :

Example Problem-1:

Construct NFA without ϵ -moves for the given NFA with ϵ -moves

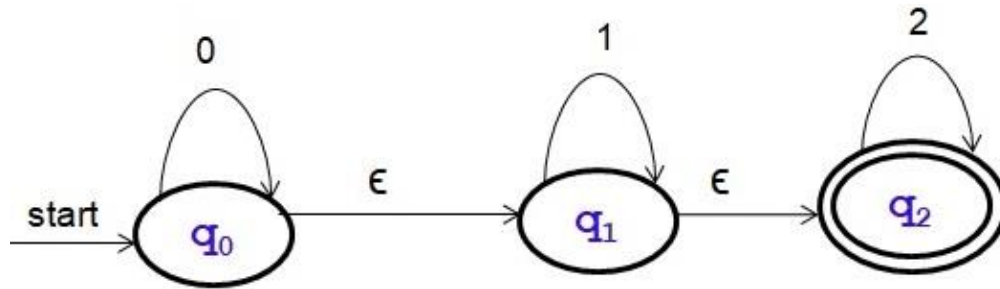
(or)

Remove ϵ -moves for the given NFA with ϵ -moves



Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



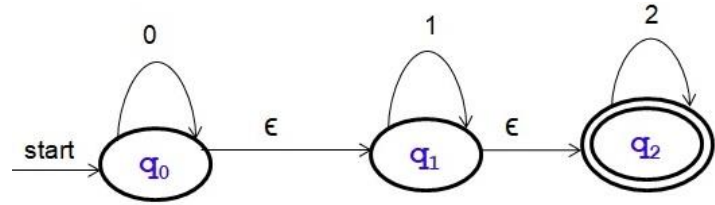
$$\delta^{\wedge}(q_0, \epsilon) = \epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\delta^{\wedge}(q_1, \epsilon) = \epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\delta^{\wedge}(q_2, \epsilon) = \epsilon\text{-closure}(q_2) = \{q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



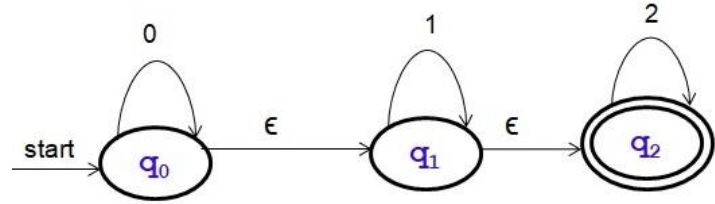
$M' = (Q, \Sigma, \delta', q_0, F')$ be an equivalent NFA without ϵ -moves

where $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1, 2\}$, $F = \{q_2\}$, q_0 is initial or start state

$$F' = F \cup \{q_0\} = \{q_2\} \cup \{q_0\} = \{q_0, q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



1.

$$\delta'(q_0, 0) = \delta^{\wedge}(q_0, 0)$$

$$= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 0))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0))$$

$$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$$

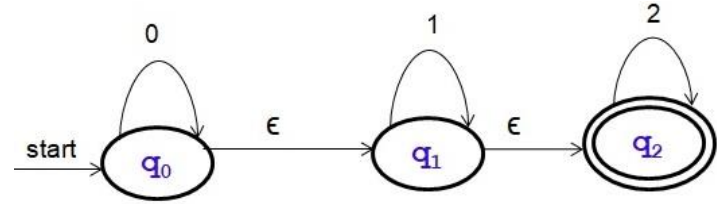
$$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi)$$

$$= \epsilon\text{-closure}(q_0)$$

$$= \{q_0, q_1, q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



$$\delta'(q_0, 1) = \delta^{\wedge}(q_0, 1)$$

$$= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 1))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 1))$$

$$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$$

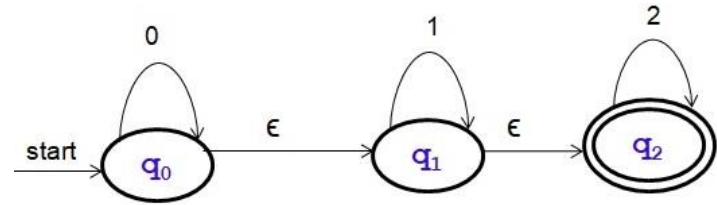
$$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$$

$$= \epsilon\text{-closure}(q_1)$$

$$= \{q_1, q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



$$\delta'(q_0, 2) = \delta^{\wedge}(q_0, 2)$$

$$= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 2))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2))$$

$$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$$

$$= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup \{q_2\})$$

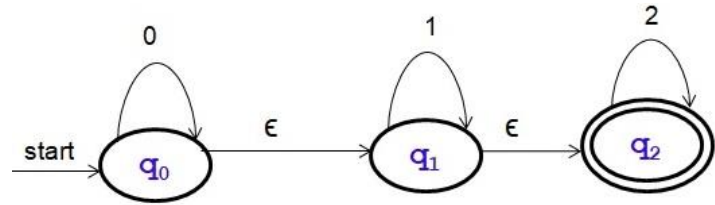
$$= \epsilon\text{-closure}(q_2)$$

$$= \{q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution

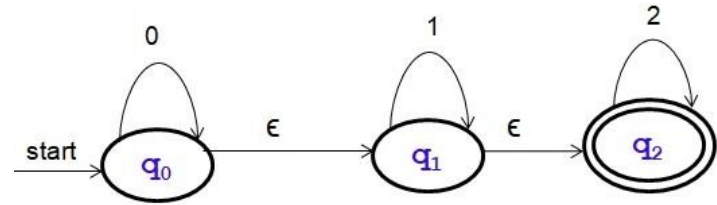
2.



$$\begin{aligned}\delta'(q_1, 0) &= \delta^{\wedge}(q_1, 0) \\ &= \epsilon - \text{closure}(\delta(\delta^{\wedge}(q_1, \epsilon), 0)) \\ &= \epsilon - \text{closure}(\delta(\{q_1, q_2\}, 0)) \\ &= \epsilon - \text{closure}(\delta(q_1, 0) \cup \delta(q_2, 0)) \\ &= \epsilon - \text{closure}(\phi \cup \phi) \\ &= \epsilon - \text{closure}(\phi) \\ &= \phi\end{aligned}$$

Equivalence of NFA with and without ϵ -moves :

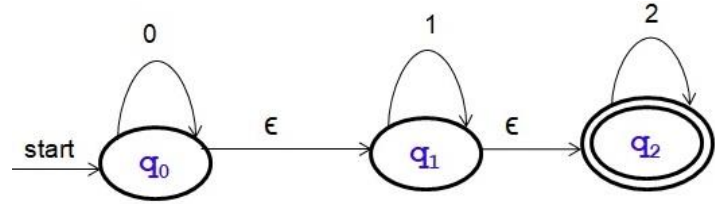
Example Problem-1: Solution



$$\begin{aligned}\delta'(q_1, 1) &= \delta^{\wedge}(q_1, 1) \\ &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_1, \epsilon), 1)) \\ &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 1)) \\ &= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1)) \\ &= \epsilon\text{-closure}(\{q_1\} \cup \phi) \\ &= \epsilon\text{-closure}(q_1) \\ &= \{q_1, q_2\}\end{aligned}$$

Equivalence of NFA with and without ϵ -moves :

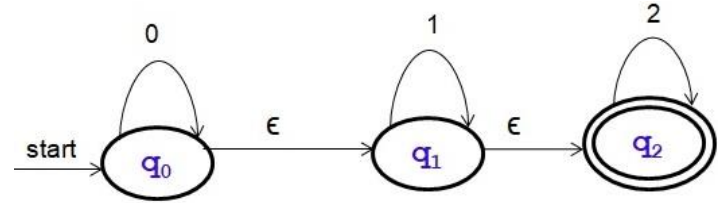
Example Problem-1: Solution



$$\begin{aligned}\delta'(q_1, 2) &= \delta^{\wedge}(q_1, 2) \\ &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_1, \epsilon), 2)) \\ &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 2)) \\ &= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2)) \\ &= \epsilon\text{-closure}(\phi \cup \{q_2\}) \\ &= \epsilon\text{-closure}(q_2) \\ &= \{q_2\}\end{aligned}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution

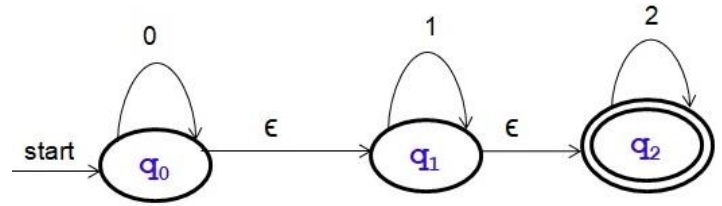


3.

$$\begin{aligned}\delta'(q_2, 0) &= \delta^{\wedge}(q_2, 0) \\ &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_2, \epsilon), 0)) \\ &= \epsilon\text{-closure}(\delta(\{q_2\}, 0)) \\ &= \epsilon\text{-closure}(\phi) \\ &= \phi\end{aligned}$$

Equivalence of NFA with and without ϵ -moves :

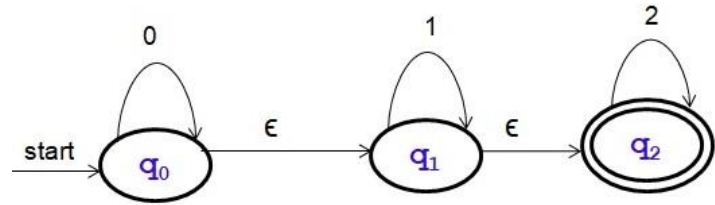
Example Problem-1: Solution



$$\begin{aligned}\delta'(q_2, 1) &= \delta^{\wedge}(q_2, 1) \\ &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_2, \epsilon), 1)) \\ &= \epsilon\text{-closure}(\delta(\{q_2\}, 1)) \\ &= \epsilon\text{-closure}(\phi) \\ &= \phi\end{aligned}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



$$\delta'(q_2, 2) = \delta^{\wedge}(q_2, 2)$$

$$= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_2, \epsilon), 2))$$

$$= \epsilon\text{-closure}(\delta(\{q_2\}, 2))$$

$$= \epsilon\text{-closure}(q_2)$$

$$= \{q_2\}$$

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution

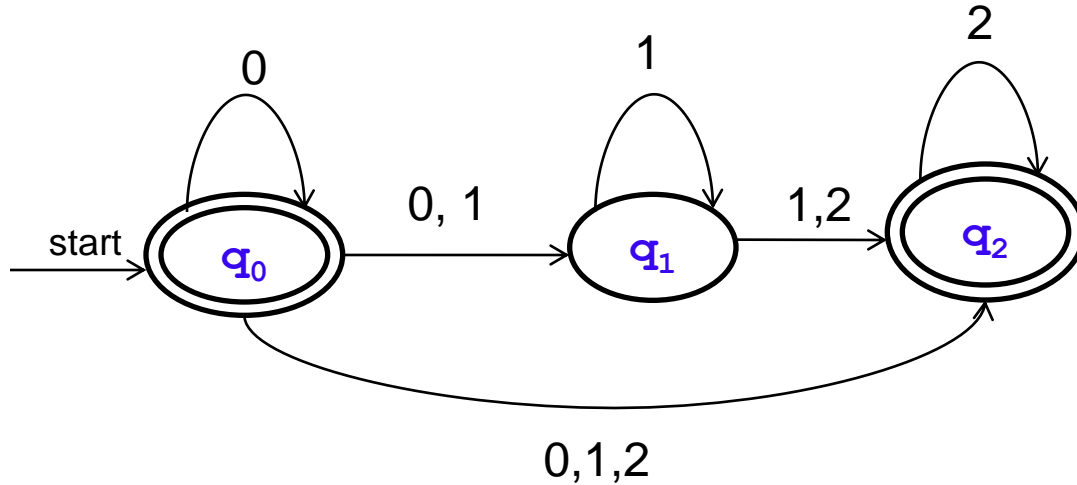
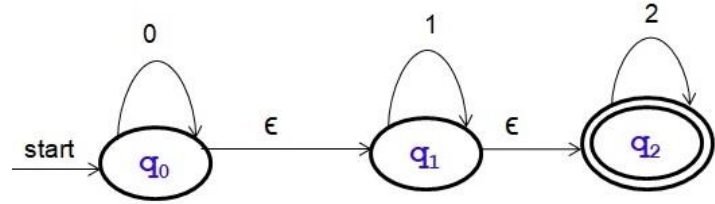
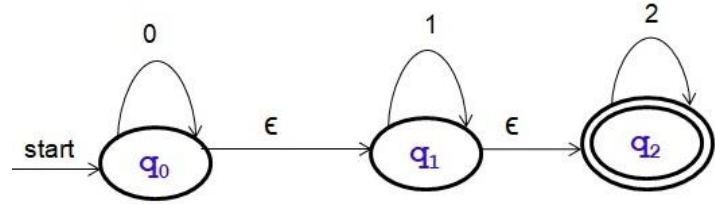


Fig: Transition Diagram of equivalent NFA without ϵ -moves

Equivalence of NFA with and without ϵ -moves :

Example Problem-1: Solution



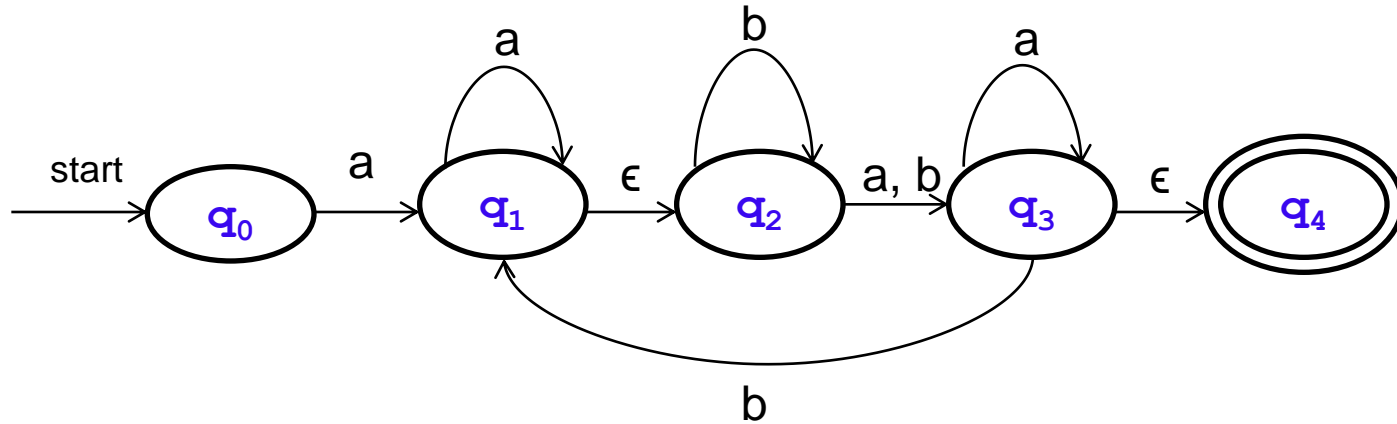
Q/ Σ	Input		
	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

Fig: Transition Table of equivalent NFA without ϵ -moves

Equivalence of NFA with and without ϵ -moves :

Practice Problem-1:

Construct NFA without ϵ -moves for the given NFA with ϵ -moves



Lecture- 08
Eliminating ϵ -transitions
Summary

- Eliminating ϵ -transitions (Equivalence of NFA with and without ϵ -moves)
- Example problem with solution