## Unit - I | Lecture- 07

# Finite Automata with Epsilon-Transitions(NFA-ϵ)

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# Finite Automata with Epsilon-Transitions(NFA-ϵ)

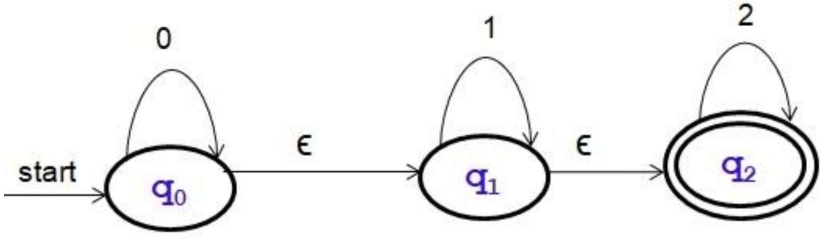
- Uses of ϵ-Transitions
- The Formal Notation for an ϵ-NFA
- Epsilon-Closures
- Extended Transitions and Languages for ϵ-NFA's
- Acceptance of Strings and Languages by ϵ-NFA

# Finite Automata with Epsilon-Transitions:
## NFA-ϵ or ϵ-NFA or NFA-E or NFA-l^

- In **NFA** the **ϵ-transitions** are given in order to move from one state to another state without having any input symbol (i.e., with empty string) from input alphabet **Σ**.

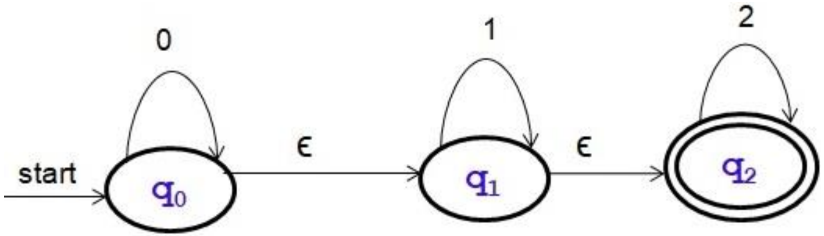
## Finite Automata with Epsilon-Transitions:
## NFA-ϵ or ϵ-NFA or NFA-E or NFA-l^

- Consider the **NFA-ϵ** given below:



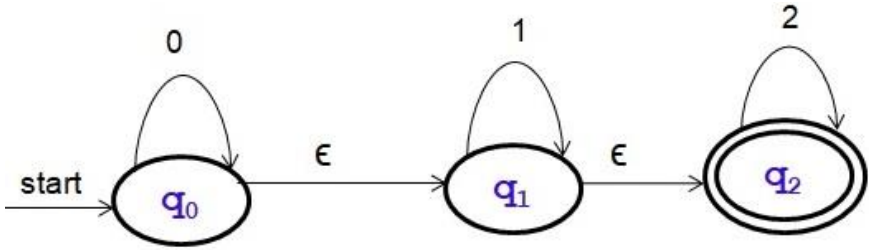

- In this **NFA-ϵ**, $q_0$ is start or initial state, $q_2$ is final or accepting state.

# Finite Automata with Epsilon-Transitions:
## NFA-ϵ or ϵ-NFA or NFA-E or NFA-l^




If we want to find an input string starting with '**1**', then we can change state from $q_0$ to $q_1$ with **ϵ-move** and then with that input we can be in state $q_1$, i.e., it is not defined with an input '**1**' whether it will be in state $q_1$ or not. Hence it is called **nondeterministic finite automata** (**NFA**) and there are some **ϵ-moves** by which we can simply change the states from one state to another state. Hence it is called **NFA** with **ϵ-transitions (NFA-ϵ)**.

# Finite Automata with Epsilon-Transitions:
## NFA-ϵ or ϵ-NFA or NFA-E or NFA-l^




In this **NFA-ϵ**, the transitions are given below:

$\delta(q_0, 0) = q_0$

$\delta(q_0, \epsilon) = q_1$

$\delta(q_1, 1) = q_1$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, 2) = q_2$

# Formal or Mathematical Definition of NFA-ϵ:

A **NFA-ϵ** is a Quin tuple or 5-tuple denoted by **M**.

i.e., $M = (Q, \Sigma, \delta, q_0, F)$

Where

$Q$ : Finite or non-empty set of States or Internal Sates

$\Sigma$ : Input Alphabet

$q_0$ : Initial State or Start State and $q_0$ is in $Q$, i.e. $q_0 \in Q$ (In any Automata initial or start state is only one)

$F$ : Set of Final or Accepting States, $F \subseteq Q$

## Formal or Mathematical Definition of NFA-ϵ :

**δ** : Transition function or Moving function or Mapping function.

Using this function, the next state can be determined.

Transition function is mapping from **Q X (Συ{ϵ})** to $2^Q$

i.e., $\delta : QX(\sum \cup \{\in\}) \rightarrow 2^Q$

Where $2^Q$ is power set of **Q**, the set of all states of **Q**.

The intension is that **δ(q,a)** will consists of all states **P** such that there is a transition labeled **'a'** from **q** to **P** where **'a'** is either **'ϵ'** or a symbol in **Σ**.

## Properties of δ function of NFA-ϵ (Extended Transition Function )

- The extended **δ** function is defined as

$$\delta^{\wedge} = Q X \sum^{*} \rightarrow 2^{Q} (\sum^{*} \; includes \in)$$

- **δ^(q, w)** will be all states **P** such that, one can go from **q** to **P** along with a path labeled **w** perhaps including edges labeled **ϵ**.
- The set of states reachable from a state **q** on **ϵ** can be determined by using a function known as **ϵ-closure(q)**.

# Properties of δ function of NFA-ϵ :

## Properties of $\delta^{\wedge}$

1. Property-1:

$$\delta^{\wedge}(q_0, \in) = \in -closure(q_0)$$

**ϵ-closure(q)** can be determined by combining set of states including **q** reachable with **ϵ** symbol.

2. Property-2:

For string **w** in **Σ*** and an input symbol **a** in **Σ**,

$$\delta^{\wedge}(q_0, wa) = \in -closure(\delta(\delta^{\wedge}(q_0, w), a))$$

# Properties of δ function of NFA-ϵ :

## Properties of $\delta^\wedge$

3. Property-3:

$$\delta(P, w) = \bigcup_{q_{in} P} \delta(q, w)$$

For each set of states $P \subseteq Q$

Ex.

$P = \{q_0, q_1\}$ , $w = a$

$\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a)$

## How a NFA-ϵ Processes Strings (Acceptance of Strings)

A string w is accepted by a **NFA-ϵ ,**

**M = (Q, Σ, δ, $q_0$, F)**

If $\delta^{\wedge}(q_0, w) = P$ , for some **P∈F**

This is basically the acceptability of a string by the final state.

## The Language of a NFA-ϵ (Acceptance of Languages by NFA):

A Language L is accepted by a **NFA-ϵ**,

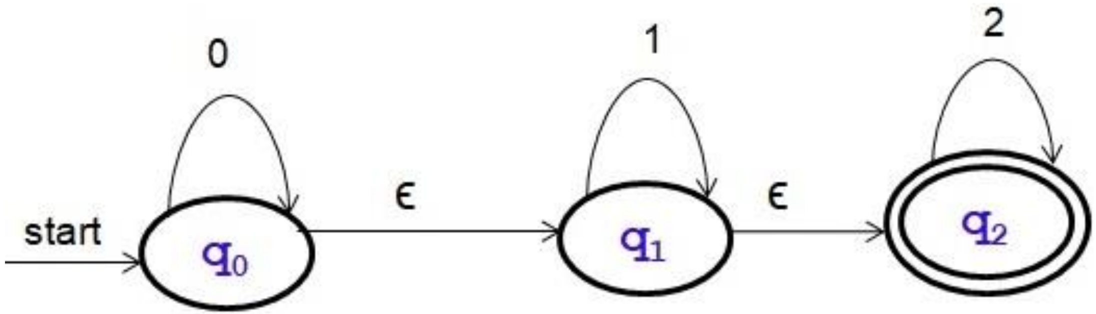
$M = (Q, \Sigma, \delta, q_0, F)$ is denoted by $L(M)$ and is the set of all strings accepted by **M**.

i.e., $L(M) = \{x | \hat{\delta}(q_0, x)$ is in some $F\}$, $x$ is a string and $x \in L$

# Acceptance of Strings by NFA-ϵ:

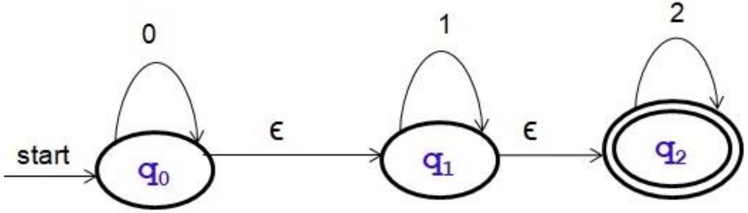
## Example Problem-1:

Consider the NFA-ϵ given below:
Check whether the input strings **01** and **012** are accepted or not by the given NFA-ϵ ?



# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution




Given input strings **01** and **012**

We needs to find $\hat{\delta}(q_0, 01)$ and $\hat{\delta}(q_0, 012)$

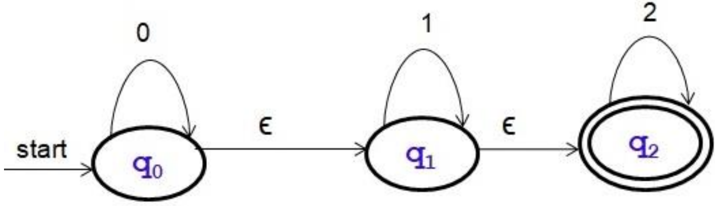
$\hat{\delta}(q_0, \epsilon)$ = ϵ-closure($q_0$) = $\{q_0, q_1, q_2\}$

$\hat{\delta}(q_1, \epsilon)$ = ϵ-closure($q_1$) = $\{q_1, q_2\}$

$\hat{\delta}(q_2, \epsilon)$ = ϵ-closure($q_2$) = $\{q_2\}$

# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution




$\delta\hat{}(q_0,\epsilon)$ = ϵ-closure($q_0$) = {$q_0$, $q_1$, $q_2$}
$\delta\hat{}(q_1,\epsilon)$ = ϵ-closure($q_1$) = {$q_1$, $q_2$}
$\delta\hat{}(q_2,\epsilon)$ = ϵ-closure($q_2$) = {$q_2$}

**I. $\delta\hat{}(q_0,01)$= ?**

$$1.\delta\hat{}(q_0,0) = \epsilon-closure(\delta(\delta\hat{}(q_0,\epsilon),0))$$

$$= \epsilon-closure(\delta(\{q_0,q_1,q_2\},0))$$

$$= \epsilon-closure(\delta(q_0,0)\cup\delta(q_1,0)\cup\delta(q_2,0))$$

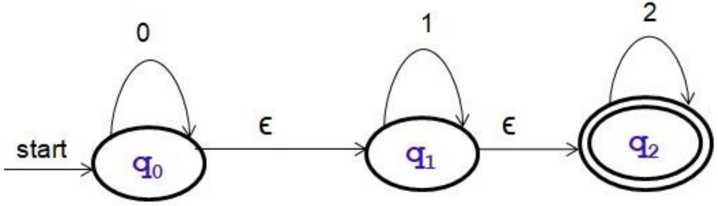
$$= \epsilon-closure(\{q_0\}\cup\phi\cup\phi)$$

$$= \epsilon-closure(q_0)$$

$$= \{q_0,q_1,q_2\}$$

# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution




$\delta\text{^}(q_0,\epsilon)$ = ϵ-closure($q_0$) = {$q_0$, $q_1$, $q_2$}
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$\delta\text{^}(q_2,\epsilon)$ = ϵ-closure($q_2$) = {$q_2$}

**$\delta\text{^}(q_0,01)$= ?**

$$2.\delta^{\wedge}(q_0,01) = \in -closure(\delta(\delta^{\wedge}(q_0,0),1))$$

$$= \in -closure(\delta(\{q_0,q_1,q_2\},1))$$

$$= \in -closure(\delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_2,1))$$

$$= \in -closure(\phi \cup \{q_1\} \cup \phi)$$

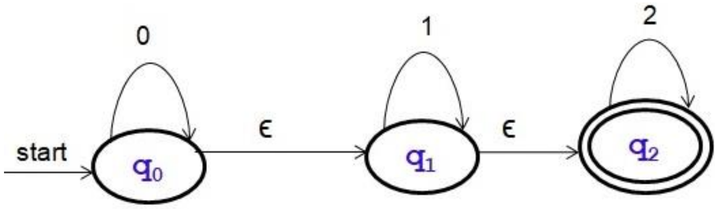
$$= \in -closure(q_1)$$

$$= \{q_1,q_2\}, q_2 \in F$$

**Conclusion:**
The input string **01** is accepted by the given NFA-ϵ.

# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution

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**II. $\delta\hat{}(q_0,012)$= ?**

$$1.\delta\hat{}(q_0,0) = \epsilon-closure(\delta(\delta\hat{}(q_0,\epsilon),0))$$

$$= \epsilon-closure(\delta(\{q_0,q_1,q_2\},0))$$

$$= \epsilon-closure(\delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_2,0))$$

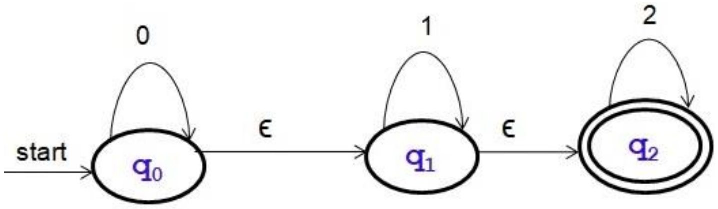
$$= \epsilon-closure(\{q_0\} \cup \phi \cup \phi)$$

$$= \epsilon-closure(q_0)$$

$$= \{q_0,q_1,q_2\}$$

# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution

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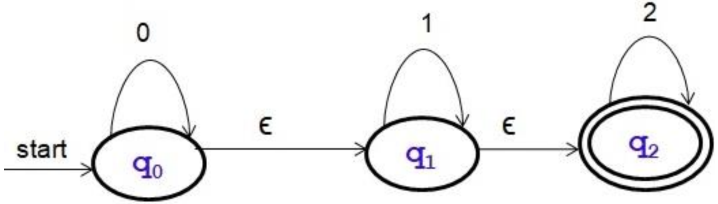
$$2.\delta^{\wedge}(q_0,01) = \in -closure(\delta(\delta^{\wedge}(q_0,0),1))$$
$$= \in -closure(\delta(\{q_0,q_1,q_2\},1))$$
$$= \in -closure(\delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_2,1))$$
$$= \in -closure(\phi \cup \{q_1\} \cup \phi)$$
$$= \in -closure(q_1)$$
$$= \{q_1,q_2\}$$

# Acceptance of Strings by NFA-ϵ:

## Example Problem-1: Solution

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$\delta\hat{}(q_2,\epsilon)$ = ϵ-closure($q_2$) = $\{q_2\}$

**II. $\delta\hat{}(q_0,012)$= ?**

$$3.\delta\hat{}(q_0,012) = \epsilon-closure(\delta(\delta\hat{}(q_0,01),2))$$
$$= \epsilon-closure(\delta(\{q_1,q_2\},2))$$
$$= \epsilon-closure(\delta(q_1,2) \cup \delta(q_2,2))$$
$$= \epsilon-closure(\phi \cup \{q_2\})$$
$$= \epsilon-closure(q_2)$$
$$= \{q_2\}, q_2 \in F$$

**Conclusion:**
The input string **012** is accepted by the given NFA-ϵ.

## Lecture- 07
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## Summary

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- Acceptance of Strings and Languages by ϵ-NFA